THE GEORGE WASHINGTON UNIVERSITY

WASHINGTON, DC

# 5a. Functional Dependencies

#### CSCI 2541W Database Systems & Team Projects

Gabe

Slides adapted from Profs. Bhagi Narahari, Tim Wood, and Rahul Simha, and book by Silberschatz, Korth, and Sudarshan

#### Good and Bad **Schemas**

**Functional Dependencies** **Normal Forms based on Functional Dependencies**

# Normal Form Examples: 1NF

**1NF:** Attributes should be atomic and tables should have no repeating groups





# Normal Form Examples: 2NF

**2NF:** No value in a table should be dependent on only **part** of a key that uniquely identifies a row



#### **VS**



1NF





# Normal Form Examples: 3NF

**3NF:** No value should be able to be dependent on another non-key field



**BAD:** Age is based on **Birthday** (non-key)



### **Summary**

1NF: ensures atomicity of cells and prevents repetition of identical column types

2NF: prevents data across rows

3NF: prevents repetition of data within+across rows

### Dependencies

#### How can we **formally represent dependencies** between Attributes in a Relation?



### Functional Dependencies

Use **functional dependencies!** (abbreviated **FD**)

We say a set of attributes **X** functionally determines an attribute **Y** if *given the values of X we always know the only possible value of Y*.

- ﹘ Notation: **X → Y**
- ﹘ X **functionally determines** Y
- ﹘ Y is **functionally dependent** on X

Example:

- $-$  GWID  $\rightarrow$  Name
- ${GWD, CourselD, Semester, Year} \rightarrow Grade$

#### Sets of Functional Dependencies

Some more functional dependencies

- $-$  {GWID}  $\rightarrow$  {NAME, ADDRESS, MAJOR}
- $-$  {MAJOR}  $\rightarrow$  {DEPT\_NAME, DEPT\_CHAIR}

From above dependencies, we can infer  $-$  {GWID}  $\rightarrow$  {DEPT\_NAME, DEPT\_CHAIR}

We can do math on functional dependencies!

A functional dependency "holds" if it must be true for all legal relations

**Armstrong's Axioms:** where A, B, C are sets of attributes

- **Reflexive rule:** if  $B \subseteq A$ , then  $A \rightarrow B$  (if B is subset of A)
- ﹘ **Augmentation rule**: if A → B, then C∪A → C∪B
- **Transitivity rule:** if  $A \rightarrow B$ , and  $B \rightarrow C$ , then  $A \rightarrow C$

These rules are

﹘ Sound and complete — generate all functional dependencies that hold.  $\{GWID\} \rightarrow \{Name, Address, Major\}$  ${Major} \rightarrow {Dept_name}$ , Dept\_Chair}  ${GWD, CourselD, Semester, Year} \rightarrow Grade$ 

**Armstrong's Axioms:** where A, B, C are sets of attributes

- $\blacksquare$  **Reflexive rule:** if  $B \subseteq A$ , then  $A \rightarrow B$  (if B is subset of A)
- ﹘ **Augmentation rule**: if A → B, then C∪A → C∪B
- **Transitivity rule**: if  $A \rightarrow B$ , and  $B \rightarrow C$ , then  $A \rightarrow C$

*Given:*

 $\{GWID\} \rightarrow \{Nam$  $[Major] \rightarrow [Dent \; Name \; Dent \; Chair]$  $VID.$ {GWID} → {Name, Address, Major} {Major} → {Dept\_Name, Dept\_Chair} {GWID, CourseID, Semester, Year} → Grade

R:{GWID, CourseID, Semester, Year} → {GWID, Year} A:{Major, Name, Address} →

 {Dept\_Name, Dept\_Chair, Name, Address} T:{GWID}  $\rightarrow$  {Name, Address, Major}  $\rightarrow$ {Dept\_Name, Dept\_Chair, Name, Address}

#### **Armstrong's Axioms:** where A, B, C are sets of attributes

- $\blacksquare$  **Reflexive rule:** if B ⊆ A, then A  $\rightarrow$  B (if B is subset of A)
- ﹘ **Augmentation rule**: if A → B, then C∪A → C∪B
- **Transitivity rule:** if  $A \rightarrow B$ , and  $B \rightarrow C$ , then  $A \rightarrow C$

These rules are

﹘ Sound and complete — generate all functional dependencies that hold.

Bonus rules to make life easier:

- $\blacksquare$  **Union rule**: If A  $\rightarrow$  B holds and A  $\rightarrow$  C holds, then A  $\rightarrow$  BUC holds.
- ﹘ **Decomposition rule**: If A → B∪C holds, then A → B holds and  $A \rightarrow C$  holds.
- ﹘ **Pseudotransitivity rule**:If A → B holds and C∪B → D holds, then  $A\cup C \rightarrow D$  holds.

#### **Armstrong's Axioms:** where A, B, C are sets of attributes

- **Reflexive rule: if B ⊆**
- **△ Augmentation rule**:
- **Transitivity rule: if A**

These rules are

- Sound and complete hold.

From now on: will *shorthand* B∪C as BC

Bonus rules to make life easier:

- $\blacksquare$  **Union rule**: If A  $\rightarrow$  B holds and A  $\rightarrow$  C holds, then A  $\rightarrow$  BUC holds.
- ﹘ **Decomposition rule**: If A → B∪C holds, then A → B holds and  $A \rightarrow C$  holds.
- ﹘ **Pseudotransitivity rule**:If A → B holds and C∪B → D holds, then  $A\cup C \rightarrow D$  holds.

#### Definition: Closure of a Set of FD's

Defn. Let F be a set of FD's. Its **closure**, **F+**, is the set of all FD's:

#### ${X \rightarrow Y \mid X \rightarrow Y}$  is derivable from F by **Armstrong's Axioms}**

**Two sets of dependencies F and G are equivalent if F+=G+** 

- i.e., their closures are equal
- i.e., the same sets of FDs can be inferred from each

#### What FDs can we infer?

 $R = (A, B, C, G, H, I)$  $F = \{ A \rightarrow B$  $A \rightarrow C$  $CG \rightarrow H$  $CG \rightarrow I$  $B \rightarrow H$ 

**Reflexive rule:** if  $\beta \subseteq \alpha$ , then  $\alpha \rightarrow \beta$ 

**Augmentation rule**: if  $\alpha \rightarrow \beta$ , then  $\gamma \alpha \rightarrow \gamma \beta$ 

**Transitivity rule:** if  $\alpha \rightarrow \beta$ , and  $\beta \rightarrow \gamma$ , then  $\alpha \rightarrow \gamma$ 

 $R = (A, B, C, G, H, I)$  $F = \{ A \rightarrow B$  $A \rightarrow C$  $CG \rightarrow H$  $CG \rightarrow I$  $B \rightarrow H$ 

**Reflexive rule:** if  $\beta \subseteq \alpha$ , then  $\alpha \rightarrow \beta$ **Augmentation rule**: if  $\alpha \rightarrow \beta$ , then  $\gamma \alpha \rightarrow \gamma \beta$ **Transitivity rule**: if  $\alpha \rightarrow \beta$ , and  $\beta \rightarrow \gamma$ , then  $\alpha \rightarrow \gamma$ 

A few members of F+ include:  $- A \rightarrow H$ 

 $- AG \rightarrow I$ 

 $- CG \rightarrow HI$ 

$$
R = (A, B, C, G, H, I)
$$
  
\n
$$
F = \{A \rightarrow B
$$
  
\n
$$
A \rightarrow C
$$
  
\n
$$
CG \rightarrow H
$$
  
\n
$$
CG \rightarrow I
$$
  
\n
$$
B \rightarrow H}
$$

**Reflexive rule:** if  $\beta \subseteq \alpha$ , then  $\alpha \rightarrow \beta$ **Augmentation rule**: if  $\alpha \rightarrow \beta$ , then  $\gamma \alpha \rightarrow \gamma \beta$ **Transitivity rule**: if  $\alpha \rightarrow \beta$ , and  $\beta \rightarrow \gamma$ , then  $\alpha \rightarrow \gamma$ 

A few members of F+ include:

 $- A \rightarrow H$ 

by transitivity from  $A \rightarrow B$  and  $B \rightarrow H$ 

 $- AG \rightarrow I$ 

 $- CG \rightarrow H1$ 

$$
R = (A, B, C, G, H, I)
$$
  
\n
$$
F = \{A \rightarrow B
$$
  
\n
$$
A \rightarrow C
$$
  
\n
$$
CG \rightarrow H
$$
  
\n
$$
CG \rightarrow I
$$
  
\n
$$
B \rightarrow H}
$$

**Reflexive rule:** if  $\beta \subseteq \alpha$ , then  $\alpha \rightarrow \beta$ **Augmentation rule**: if  $\alpha \rightarrow \beta$ , then  $\gamma \alpha \rightarrow \gamma \beta$ **Transitivity rule**: if  $\alpha \rightarrow \beta$ , and  $\beta \rightarrow \gamma$ , then  $\alpha \rightarrow \gamma$ 

A few members of F+ include:

 $- A \rightarrow H$ 

by transitivity from  $A \rightarrow B$  and  $B \rightarrow H$ 

 $- AG \rightarrow I$ 

by augmenting  $A \rightarrow C$  with G, to get  $AG \rightarrow CG$ and then transitivity with  $CG \rightarrow I$ 

 $- CG \rightarrow H1$ 

```
R = (A, B, C, G, H, I)F = \{ A \rightarrow B \}A \rightarrow CCG \rightarrow HCG \rightarrow IB \rightarrow H}
```
**Reflexive rule:** if  $\beta \subseteq \alpha$ , then  $\alpha \rightarrow \beta$ **Augmentation rule**: if  $\alpha \rightarrow \beta$ , then  $\gamma \alpha \rightarrow \gamma \beta$ **Transitivity rule**: if  $\alpha \rightarrow \beta$ , and  $\beta \rightarrow \gamma$ , then  $\alpha \rightarrow \gamma$ 

A few members of F+ include:

 $- A \rightarrow H$ 

by transitivity from  $A \rightarrow B$  and  $B \rightarrow H$ 

 $- AG \rightarrow I$ 

by augmenting  $A \rightarrow C$  with G, to get  $AG \rightarrow CG$ and then transitivity with  $CG \rightarrow I$ 

 $- CG \rightarrow H1$ 

by augmenting  $CG \rightarrow I$  to infer  $CG \rightarrow CGI$  (note  $CGCG \rightarrow CGI$  is  $CG \rightarrow CGI)$ , and augmenting of  $CG \rightarrow H$  to infer CGI  $\rightarrow H$ I, and then transitivity (CG  $\rightarrow$  CGI  $\rightarrow$  HI).

This derives the *Union* rule!

#### Functional Dependencies and Keys

A Candidate Key is a minimal set of attributes which are sufficient to uniquely identify each tuple in a relation

﹘ All other attributes must be functionally dependent on the set of attributes that make up the Candidate Key.

Thus a candidate key must be a minimal set of attributes which can appear on the left hand side of functional dependencies, but will produce a closure that includes all other attributes on the right hand side

$$
\{A,\,B\}\longrightarrow\!\{C,\,D,\,E\}
$$

#### Functional Dependencies and Keys

A candidate key must be a minimal set of attributes which can appear on the left hand side of functional dependencies, but will produce a closure that includes all other attributes on the right hand side

What is a candidate key for R?

```
R = (A, B, C, G, H, I)F = \{ A \rightarrow BA \rightarrow CCG \rightarrow HCG \rightarrow IB \rightarrow H
```
#### Functional Dependencies and Keys

A candidate key must be a minimal set of attributes which can appear on the left hand side of functional dependencies, but will produce a closure that includes all other attributes on the right hand side

What is a candidate key for R?

```
R = (A, B, C, G, H, I)F = \{ A \rightarrow BA \rightarrow CCG \rightarrow HCG \rightarrow IB \rightarrow H
```
Candidate Key: (A, G) Non-Prime Attribs: (B,C,G,H,I)

#### **Functional Department**

A candidate key must b appear on the left hand produce a closure that hand side

What is a candidate key

 $R = (A, B, C, G, H, I)$  $F = \{ A \rightarrow B$  $A \rightarrow C$  $CG \rightarrow H$  $CG \rightarrow I$  $B \rightarrow H$ 

#### **Pseudotransitivity rule**:

If  $A \rightarrow B$  holds and  $CB \rightarrow D$  holds, then  $AC \rightarrow D$  holds.

How? A→C and CG→H Augmentation: AG→CG Transitivity: AG →CG→H

> Candidate Key: (A, G) Non-Prime Attribs: (B,C,G,H,I)

#### Good and Bad **Schemas**

**Functional Dependencies** **Normal Forms based on Functional Dependencies**

#### Redefining 2NF No value in a table should be dependent on only **part** of a key that uniquely identifies a row

Using Functional Dependencies and Closures lets us more precisely define our Normal Forms

**Second Normal Form:** For every  $X \rightarrow A$  that holds over relationship schema **R**, where A is a non-prime attribute

- (i.e., A is not an attribute in any candidate key)
- 1. either **A** ∈ **X** (it is trivial), or
- 2. **X** is a superkey for **R**, or
- 3. **X** is transitively dependent on a super key R

*Easier to think of the opposite*: There cannot be **X → A**  where X is a partial candidate key for R

Says nothing about non-prime to non-prime dependencies!

# 2NF Violations





# 2NF Violations



**ID** -> {**First Name, LastName**} Violates 2NF since **ID** is a partial Candidate Key



#### No 2NF violation

# Redefining 3NF

Third Normal Form (3NF): For every  $X \rightarrow A$  that holds over relationship schema **R**,

- 1. either **A** ∈ **X** (it is trivial), or
- 2. **X** is a superkey for **R**, or
- 3. **A** is a member of some key for **R**

*Easier to think of*: **X** must be a full candidate key, unless A itself is a part of a candidate key

"Every non-key attribute must provide a fact about the Key, the whole Key, and nothing but the Key... so help me Codd"





### 3NF Violations

 $C \rightarrow$ F,S,B,A,F



#### **Birthday->Age** holds, but **Birthday** is not a superkey

 $T, Y \rightarrow W$ , WB  $W \rightarrow WB$ 



**Winner -> Birthplace** holds, but **Winner** is not a superkey

### Normal Forms 1-3

**1NF**: Attributes should be atomic and tables should have no repeating groups

- *Prevents messiness within a cell and repetition of rows*
- **2NF:** There cannot be  $X \rightarrow A$  where X is a partial candidate key for R
	- Doesn't forbid non-prime to non-prime dependencies
	- *Prevents repetition of cells across rows*

**3NF:** There cannot be  $X \rightarrow A$  where X is not a full candidate key for R (unless A is a Key)

- ﹘ Only allows dependencies on Keys
- *Prevents repetition of data within a row*

#### Good and Bad **Schemas**

#### **Functional Dependencies**

#### **Even more normal forms!**

### Normal Forms 1-3

**1NF**: Attributes should be atomic and tables should have no repeating groups

- *Prevents messiness within a cell and repetition of rows*
- **2NF**: There cannot be  $X \rightarrow A$  where X is a partial candidate key for R
	- Doesn't forbid non-prime to non-prime dependencies
	- *Prevents repetition of cells across rows*
- **3NF:** There cannot be  $X \rightarrow A$  where X is not a full candidate key for R (unless A is a Key)
	- Only allows dependencies on Keys
	- *Prevents repetition of data within a row*

### Normal Forms 1-3

**1NF**: Attributes should be atomic and tables should have no re - Prevent *Prevent* Preview of the Isometimes of rows **2NF**: There can be a set of  $\alpha$  **artial** candidate — Doesn't forbid non-prime to non-prime to non-prime to the non-prime dependence of  $\mathbb{R}^n$  *Prevents repetition of cells across rows* **3NF**: There **cannot be a cannot be a full** candidate  $-$  Only all *Prevents repetition of data within a row* Preview of the (sometimes unrealistic) goal:  $R = (A, B, C, D, E, F)$ has FD+: ABC **→** DEF

# Normal Form

Normal form reference:

- ﹘ 2NF: Cannot have partial Key on left hand side (LHS)
- ﹘ 3NF: Meet 2NF and LHS must be full Candidate Key or RHS must be a key



**Functional Dependencies**

 $ID \rightarrow FirstName$ ID, Cid → Num, Grade Num → Subj

What normal form is this?

# Normal Form

Normal form reference:

- ﹘ 2NF: Cannot have partial Key on left hand side (LHS)
- ﹘ 3NF: Meet 2NF and LHS must be full Candidate Key or RHS must be a key



**Functional Dependencies**

ID → FirstName partial key ID violates 2NF! ID, Cid → Num, Grade Num → Subj non-prime LHS would also violate 3NF!

#### Only meets 1NF

# How to Judge Decomposition?



ID  $\rightarrow$  FirstName ID, Cid → Num, Grade  $Num \rightarrow Subj$ 

Lossless Decomposition test:

- ﹘ **R1**, **R2** is a lossless join decomposition of **R** with respect to **F iff** at least one of the following dependencies is in **F+**
- ﹘ **(R1 ∩ R2) → R1 R2**
- ﹘ **(R1 ∩ R2) → R2 R1**

### Lossless Decomposition



- ﹘ **R1**, **R2** is a lossless join decomposition of **R** with respect to **F iff** at least one of the following dependencies is in **F+**
- ﹘ **(R1 ∩ R2) → R1 R2**
- ﹘ **(R1 ∩ R2) → R2 R1**

### Lossless Decomposition



﹘ **(R1 ∩ R2) → R2 – R1**

### Dependency Preservation

We also must maintain dependences

After decomposition from **R** to **R1 … Rn**, the closure of FDs of all **R1…Rn** must be equivalent to that of **R**

**R1 = ID,** FirstName, CID **R2 = CID,** Sub, Num, Grade

or

ID → FirstName ID, Cid → Num, Grade Num → Subj

 $R3 = 1D$ , FirstName **R4 = ID, CID,** Sub, Num, Grade

### Dependency Preservation

We also must maintain dependences

After decomposition from **R** to **R1 … Rn**, the closure of FDs of all **R1…Rn** must be equivalent to that of **R**

**R1 = ID,** FirstName, CID **R2 = CID,** Sub, Num, Grade

or

 $R1, R2$  will lose the FD:  $\|$  Num  $\rightarrow$  Subj ID,CID -> Num, Grade

**R3 = ID,** FirstName **R4 = ID, CID,** Sub, Num, Grade

ID → FirstName ID, Cid → Num, Grade

> R3,R4 will maintain all FDs (why Sub?)

### 3NF

It is **always possible** to decompose a relation R into a set of relations R1…Rn which is **dependency preserving** and **lossless** that is in 3NF

> 3NF is the baseline for acceptable DB normalization in practice! Required!

> > but 3NF is not perfect…

### When does 3NF fail?

Suppose we want to store addresses:



#### Meets 3NF since LHS is a full Key **or** RHS is a Key

3NF: There cannot be  $X \rightarrow A$  where X is not a full candidate key for R **(unless A is a Key)**

#### When does 3NF fail?

#### ADDR\_INFO( **CITY**, **ADDRESS**, **ZIP**)  $\{CITY, ADDRESS\} \rightarrow ZIP$  ${ZIP} \rightarrow {CITY}$



3NF: There cannot be  $X \rightarrow A$  where X is not a full candidate key for R **(unless A is a Key)**

#### When does 3NF fail?

#### ADDR\_INFO( **CITY**, **ADDRESS**, **ZIP**)  $\{CITY, ADDRESS\} \rightarrow ZIP$  ${ZIP} \rightarrow {CITY}$



#### 3NF does not prevent insertion/update of tuples which violate our FDs!

# 3NF vs BCNF

Third Normal Form (3NF): For every  $X \rightarrow A$  that holds over relationship schema **R**,

- 1. either  $A \in X$  (it is trivial), or
- 2. **X** is a superkey for **R**, or
- 3. **A** is a member of some key for **R**

Option 3 can result in update anomalies!

#### **Boyce-Codd Normal Form** (BCNF) resolves this issue:

For every  $X \rightarrow A$  that holds over relationship schema R,

- 1. either  $A \in X$  (it is trivial), or
- 2. **X** is a superkey for **R**

### 3NF vs BCNF



# BCNF

#### BCNF is stricter than 3NF

- ﹘ If a relation is in BCNF, it is also in 3NF;
- ﹘ if it is not in 3NF, it is not in BCNF

Note:

- ﹘ There are polynomial time algorithms **guaranteed to provide a lossless, dependency preserving decomposition** into 3NF
- ﹘ **but** a **dependency preserving** decomposition into BCNF **may not exist**, and no polynomial time algorithm for **lossless decomposition** is known.

### Normalization Summary

**Functional Dependencies**: Capture the dependencies between attributes

**Normalization**: Provides a schema that ensures functional dependencies will be kept consistent, without losing data

**Normal Forms**: Try to achieve BCNF, but 3NF is OK in some cases (1NF/2NF -> bad design!)

### 2NF vs 3NF vs BCNF

Second Normal Form (2NF): For every  $X \rightarrow A$  that holds over relationship schema **R**,

1. If **A** is a non-prime attribute, then **X** cannot be a partial Candidate Key

Third Normal Form (3NF): For every  $X \rightarrow A$  that holds over relationship schema **R**,

- 1. either  $A \subseteq X$  (it is trivial), or
- 2. **X** is a superkey for **R**, or
- 3. **A** is a member of some key for **R**

Option 3 can result in update anomalies!

#### **Boyce-Codd Normal Form** (BCNF) resolves this issue:

For every  $X \rightarrow A$  that holds over relationship schema R,

1. either  $A \subseteq X$  (it is trivial), or

2. **X** is a superkey for **R**